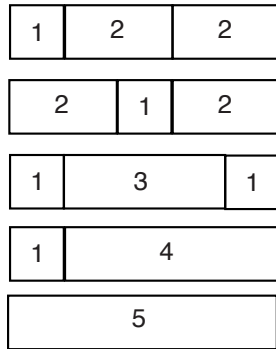


**Idea 2: Trains.** You can use rods of whole number sizes to build “trains” that all share a common length. A “train of length 5” is a row of rods whose combined length is 5. Here are some examples



Notice that the 1-2-2 train and the 2-1-2 train contain the same rods but are listed separately. If you use identical rods in a different order, this is a separate train.

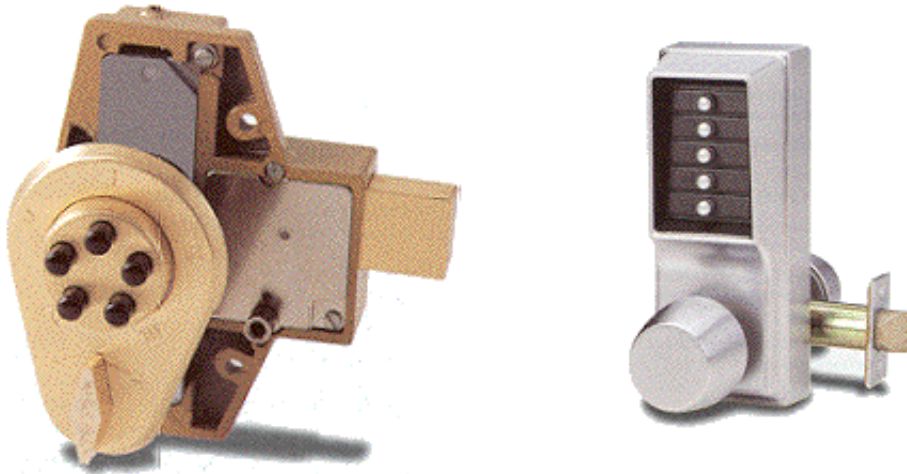
3. How many trains of length 5 are there?
  4. How many trains of length 6 are there?
  5. How many trains of length 10 are there?
  6. Come up with a formula for the number of trains of any length. (Assume you have rods of every possible integer length available.) Explain your result.
- Will a graph help? A table?

**Project ideas:**

1. *Idea 1:* How many trains of length 5 have:
  - one car?
  - two cars?
  - three cars?
  - four cars?
  - five cars?
2. *Idea 2:* What if you could only use trains of length 1 and 2?

These are just suggestions. You can make up train projects of your own.

**Idea 3: The Simplex Lock.** The Simplex company makes a combination lock that is used in many public buildings. It comes in several versions. Here are two:



These 5-button devices are purely mechanical (no electronics). You can set the combination using the following rules:

1. A combination is a sequence of 0 or more pushes, each push involving at least one button.
2. Each button may be used at most once (once you press it, it stays in).
3. Each push may include any number of “open” buttons, from one to five.
4. When two or more buttons are pushed at the same time, order doesn’t matter.

Possible combinations:

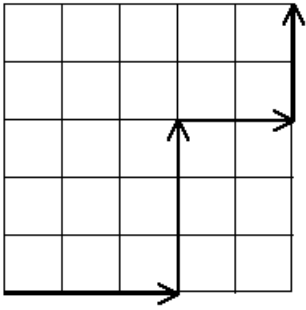
- $\{\{1, 2\}, \{3\}\}$
- $\{\{1, 2, 4\}, \{3, 5\}\}$
- $\{\{3\}, \{1, 2\}\}$
- $\{\{2, 1\}, \{3\}\}$
- $\{\{1, 2, 4, 3, 5\}\}$
- $\{\}$
- $\{\{2\}, \{1\}, \{3\}\}$
- $\{\{1, 2\}, \{4\}, \{3, 5\}\}$
- $\{2\}$

Notation:  $\{\{1, 2\}, \{3\}\}$  means “press 1 and 2 together, then press 3.”

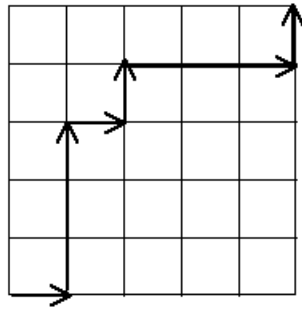
The company advertises thousands of combinations, and, the question is, “How many combinations are there? Is the company telling the truth?”

**Project idea:** What if the lock had 6 buttons? 7 buttons?

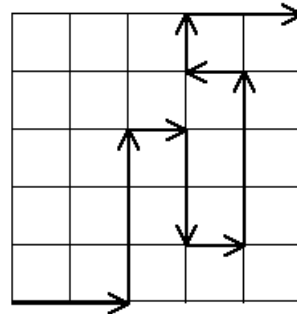
**Idea 4: Paths** Ms. D'Amato likes to take a different route to work every day. She will quit her job the day she has to repeat her route. Her home and work are pictured in the grid of streets below. If she never backtracks (she only travels north or east), how many days will she work at this job?



A valid trip



Another valid trip



Not a valid trip

**Project idea:** What about different grid sizes?

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**Idea 5: Dice.** This project involves figuring out the sums that can show up when you throw dice.

7. When tossing a single die, how many different numbers can show up? What is the most likely number?
8. When tossing two dice, how many different pairs can show up? How many different sums can show up? What is the most likely sum?
9. When tossing three dice, how many different pairs can show up? How many different sums can show up? What is the most likely sum?

Of course, you may say, "Dice don't have numbers, they have dots!"

**Project ideas:** Find a way answer the questions like those above for *any* number of dice. For example,

- If 6 dice are thrown, how many pairs can show up?
- If 4 dice are thrown, what's the most likely sum?
- If 4 dice are thrown, how likely is it to roll a 9?