

## **APPENDIX 2 :**

**What is Knowledge of Mathematics for  
Teaching?**

**Does the Focus on Mathematics  
Immersion Program achieve it?  
How can we assess it?  
Early Evidence**

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As the external evaluators on the Focus on Mathematics [FoM] Math/Science Partnership, we have been documenting the development of the partnership, the implementation of their professional development programs, and collecting data from participating middle and high school mathematics teachers from Arlington, Chelsea, Lawrence, Waltham and Watertown school districts.

Last fall, we conducted a set of in-depth interviews with seven ‘mathematics teaching fellows’ who participated in last summer’s immersion program, called PROMYS for Teachers, and who have enrolled in the new masters program at Boston University. The teachers were diverse in terms of their educational background [the extent to which they had studied mathematics in undergraduate or graduate programs], the number of years they have taught mathematics, and the grade levels they currently teach. Interviewees included both middle and high school mathematics teachers. All five of the districts that are partners in the Focus on Mathematics Program were represented.

Our preliminary evidence documents that teachers in the Focus on Mathematics immersion program are deepening their knowledge of mathematics for teaching and are beginning to identify new instructional approaches based on that learning, and, in some cases, they are integrating those into their classroom practices.

Teachers’ comments on their experience in the summer program, what they gained from it, and their reports on how that experience was influencing their approach to teaching mathematics can be found following the discussion of our next steps, below.

#### Next Steps:

To study teachers’ knowledge of mathematics for teaching and how it is manifested, we are designing case studies of 6 teachers engaged in Focus on Mathematics immersion program to look how teachers involved in the program:

- Deepen their knowledge of mathematics for teaching
- Shift their approach to teaching mathematics
- Provide opportunities for students to work on mathematical problems and projects of real substance
- Influence mathematics teaching and learning in their schools and districts

With extended support we hope to

- Develop processes for observing and documenting changes in their work with students
- Develop and pilot classroom observation protocols as well as performance assessments to assess knowledge of mathematics for teaching
- Examine their efforts to influence and lead district change, and the impact of those efforts.

We welcome comments and suggestions from and collaborations with members of the MSP community.

Focus on Mathematics Teachers' Comments on the PROMYS for Teachers Summer Institute, 2004.

One of the things the mathematician threw at us was fundamental calculus. I am seeing more connections. Calculus is about rate of change. In one sense, kids at the elementary level are doing calculus, they're just not calling it that. The mathematician we are working with is showing us that all these different subjects within mathematics are tied together much more than I thought.

It [summer immersion program] was learning that discovery feeling -- with your own ability, coming up with your own conclusions. It also gave you a deep understanding of the things we teach -- the everyday things. [It was an opportunity] to delve in and understand the theory behind them, for our teaching.

I was the student now, who had totally foreign material in front of me that I needed to learn - difficult concepts. Some I couldn't get. It made me understand how my students feel when I'm teaching something and they are not grasping it. It put me in their shoes and it made me think about how I ask questions and how I learn. It made me think about how I can help my students learn to be more independent, and how to attack things that they have no idea about, where to start. It gave me a way to help them approach things they don't know.

They'd ask us a series of questions and it was up to us to take those questions and see what happened. At first it was frustrating, then we enjoyed it. No one explicitly tells us the answers. It was constructive, it encouraged us to dig deeper. I wouldn't have wanted too much support; it would have cheated the experience. I don't know that that would have helped us grow any further.

I became very sensitive to the emotional aspects of learning and math. Most math teachers love math but over the summer I was sometimes close to tears. My students also get frustrated to the point of tears. I think it is a great sign because they are feeling the emotional aspect and I think if we can turn that frustration into a level of excitement. I think PROMYS was showing clues about how to redirect those frustration points or get our students to work past those points. This summer gave me new ideas. If you want a powerful math community you have to find a way to get students hooked or have some feeling toward it and see the beauty in it. With number theory you start to see the beauty.

The first weeks of the program, I could connect to things I knew. Even if I was frustrated one day, the next day I'd have an epiphany. There were lots of ups and downs. Understanding math concepts was not enough. You had to look at things in different ways. It's not necessarily intuitive. I learned a lot about my own patience. Every time I felt frustrated I realized something I would not have realized without being frustrated.

I learned a lot about number theory and fell in love with it ... and computation and understanding of numbers, and counting. My understanding of number sense is much stronger and it is not just out of memorizing things but seeing what happens and why with numbers.

One powerful lesson I learned will help me understand how to diagnose students to determine what they know. I realized that with anything abstract given to me I couldn't work with algebraic equations but had to do numericals first. I had to prove it was true in the first place with number. Then algebraic representation came to me. Teaching algebra to grade 8 and kids that are all over place with their math development, this will really help me with those not ready for more abstract stuff. I have to give them more numerical work until they say, "I know what is going to happen."

Part of it was self-discovery. Like for me, how to attack something I have no idea about. That really helped.

A lot of us didn't feel we were prepared for the summer program. Afterwards felt we could do anything.

I'm looking forward to doing it again next summer. I like the idea of going back next summer and doing the problem sets again and work on all the things I didn't get to work on this summer. Sometimes, the connections I'd seen lasted only for a split second.

I've taken some of these proofs, some of the more accessible proofs, and incorporated them into my teaching. I tell them and myself don't accept things as established facts, wait until you see why it works.

There's no class like it. I was exposed to the new way of letting someone go, providing them with all the questions and none of the answers. At PfT, they are guiding the learning but not explicitly stating it. It showed me I can learn in a different way.

I will find places to build number sense. The first week I gave them blank multiplication tables. I asked them to do them as homework – they said no problem. Next day I said how many did not need to do out all the multiplication because they saw patterns? Hands went up. I told them, for each problem you can explain what is happening, I'll give a point on the next test. I will integrate more things like that and they love it.

I believe it already has [changed my teaching], I've been teaching more number theory embedded in Algebra. I used a whole section on place value and powers of 10 and kids liked seeing the deeper value of how division works or multiplication works. This is only my 2<sup>nd</sup> year teaching but I will continue to develop as I go through the program -- to explore more deeply certain things. I've already started this year, It's

exciting to already use this stuff and every year I could use more. Going back [to the program] next summer will help with that even more.

Some of the things I learned when I was in high school never made sense to me, but after this summer, I have an understanding of prime numbers and composite numbers that I didn't really understand before.